H. Algebra 2- Semester 1 Final Exam Review Name: ______________________

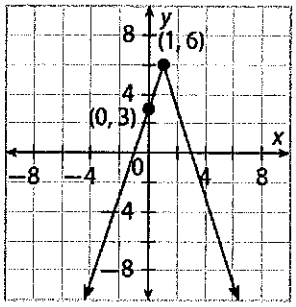
Unit 1 – Functions

Module 1 – Analyzing Functions

1.1 – Domain, Range and End Behavior

For 1-2, give the domain and range for each function in inequality, set, and interval notation.

1.



Domain

$-\infty < x < \infty$

Set

$\{x \mid -\infty < x < \infty\}$

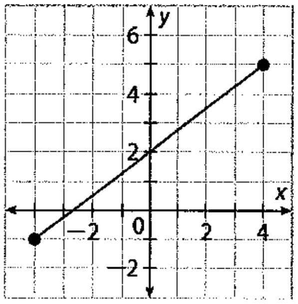
Interval

$(-\infty, \infty)$

Range

$X \le 6$ $\{y \mid y \le 3\}$ $(-\infty, 3]$

2.



Domain

$-4 \le X \le 4$

$\{x \mid -4 \le x \le 4\}$

$[-4, 4]$

Range

$-1 \le y \le 5$

$\{y \mid -1 \le y \le 5\}$

$[-1, 5]$

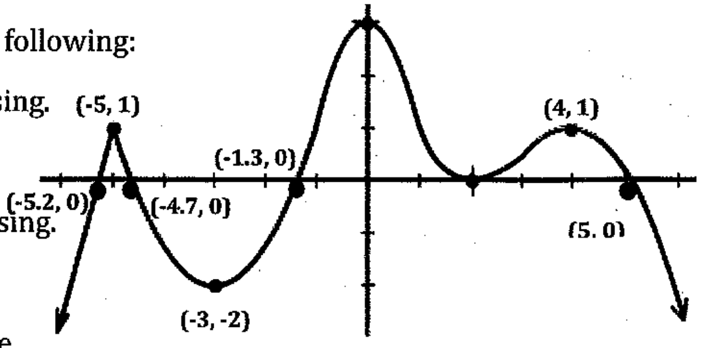
3. Describe the end behavior for the graph in problem 1.

as $x \to \infty, y \to -\infty$

as $x \to -\infty, y \to -\infty$

1.2 - Characteristics of Function Graphs

For 4 - 12, use the graph to the right to identify the following:



4. Interval(s) where the function values are increasing.

$(-\infty, -5) \cup (-3, 0) \cup (2, 4)$

5. Interval(s) where the function values are decreasing.

$(-5, -3)\ (0, 2)\ (4, \infty)$

6. Interval(s) where the function values are positive.

$(-5.2, -4.7)(-1.3, 2) \cup (2, 5)$

7. Intervals where the function values are negative.

$(-\infty, -5.2) \cup (-4.7, -1.3) \cup (5, \infty)$

8. Local minimum and local maximum values.

$(-3, -2)\ (2, 0)$ $(-5, 1)(0, 4)\ (4, 1)$

9. The zeros of the function.

$(-5.2, 0)(-4.7, 0)$

$(-1.3, 0)(2, 0)\ (5, 0)$

10. End Behavior

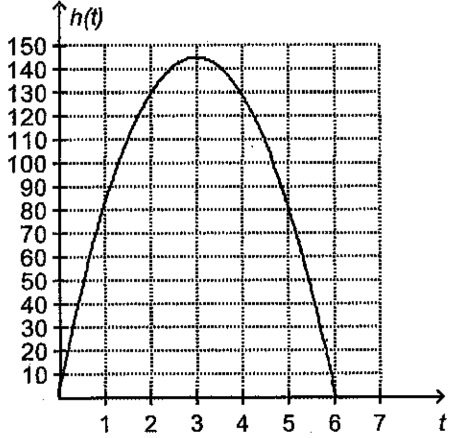
as $x \to \infty, y \to -\infty$

as $x \to -\infty, y \to -\infty$

11. The function $A(d) = 0.45d + 180$ models the amount A, in dollars, that Terry's company pays him based on the round-trip distance d, in miles, that Terry travels to a job site. How much does Terry's pay increase for every mile of travel?

$ .45 per mile

12. The graph shows the height $h(t)$ of a model rocket t seconds after it is launched from the ground at 48 feet per second.



A. What is the maximum height the rocket reaches?

145 feet

B. At what time does it reach its maximum height?

3 seconds

C. What interval of time is the height of the rocket increasing? (0,3)

D. What interval of time is the height of the rocket decreasing? (3,6)

13.

x	1	2	3	4	5
y	10	21	28	37	52

a. Use calculator to make a scatter plot.

b. Make a best fitted line and find the function of the line. y= 10x + -.4

c. Predict the value of y when x =10 99.6

d. Is your prediction from part c an interpolation or extrapolation?

1.3 – Transformations of Function Graphs

14. Given the general equation $g(x) = a \cdot f\left(\frac{1}{b}(x - h)\right) + k$, identify the effect each variable has on the graph of *f(x)*.

a = Vertical Stretches and Reflect over x axis

b = Horizonta Stretches and reflect over y axis

h = slide right or left

k = Slide up or down

For 15-18, describe in words the transformations applied to *f(x)* to obtain the graph of *g(x)*.

15. $g(x) = -\frac{2}{3}f(x+1)$ Vertical compressi fact 2/3

reflect over x-axis

1 - left

16. $g(x) = 3f(x) - 5$ 5 Down

vertical Stretch

factor of 3

17. $g(x) = f(\frac{1}{2}(x-3))$

Horizontal stretch of factor 2

3 right

18. $g(x) = f(-3x) + 1$

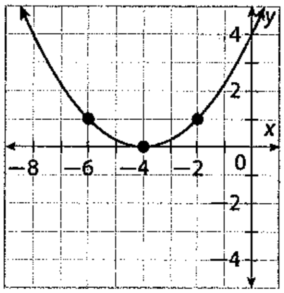
reflect over y-axis

Horizontal compression factor 1/3

1 up

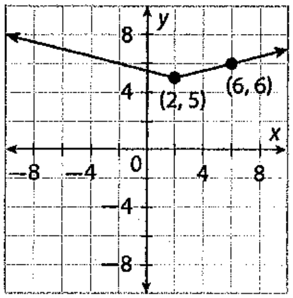
Write a function to match each graph shown.

19.



$g(x) = (\frac{1}{2}(x+4))^2$

20.



$g(x) = |\frac{1}{4}(x-2)|+5$

1.4 - Inverses of Functions

21. What is the relationship between the points on the graph of function and the points on the graph of its inverse? $(x, y) = (y, x)$

For 22-23, find the inverse for each function. Write equations in slope-intercept form.

22. $f(x) = -4x + 12$

$x = -4y + 12$

$x - 12 = -4y$

$y = -\frac{1}{4}x + 3$

$f^{-1}(x) = -\frac{1}{4}x + 3$

23. $f(x) = \frac{2}{3}x - 6$

$x = \frac{2}{3}y - 6$

$x + 6 = \frac{2}{3}y$

$y = \frac{3}{2}x + 9$

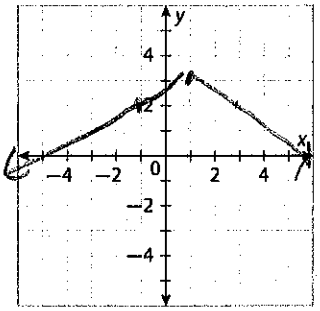
$f^{-1}(x) = \frac{3}{2}x + 9$

24. Given that the function $f(x)$ is defined by the set of point $\{(0,1), (2,5), (-4,5), \textit{and } (8,-3)\}$, what set of points defines $f^{-1}(x)$? (1,0) (5,2) (5,-4) (-3,8)

Module 2 – Absolute Value Functions, Equations, and Inequalities

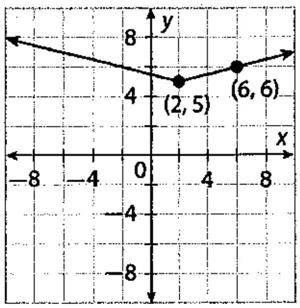
2.1- Graphing Absolute Values Functions

1. Graph the function $f(x) = -\frac{1}{2}|x-1| + 3$



2. Write a function for the graph shown.

See # 20



2.2 – Solving Absolute Value Equations

For 3-4, solve each absolute value equation algebraically. Then graph the solution(s) on the number line.

3. $|2x - 3| = 1$

$2x - 3 = 1$ $\quad$ $2x - 3 = -1$

$2x = 4$ $\quad$ $2x = 2$

$x = 2$ $\quad$ $x = 1$

4. $|x + 4| + 3 = 2$

$-3 \quad -3$

$|x+4| = -1$

no solution

$|\ |$ can't be −

Unit 2- Quadratic Functions, Relations, and Equations

Module 3 - Quadratic Equations

3.1 Solving Quadratic Equations by Taking Square Roots

For 1-3, solve by taking square roots. Tell whether each solution is real or imaginary. *Give exact answers. Express imperfect roots in simplified radical form.*

1. $2x^2 - 16 = 0$

$2x^2 = 16$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

2. $-5x^2 + 9 = 0$

$-5x^2 = -9$

$\sqrt{x^2} = \sqrt{9/5}$

$x = \pm \frac{3}{\sqrt{5}}$ or $\pm \frac{3\sqrt{5}}{5}$

3. $4x^2 = x^2 - 42$

$3x^2 = -42$

$\sqrt{x^2} = \sqrt{-14}$

$x = \pm i\sqrt{14}$?

For problems 4-5, recall the equation for falling objects:

4. A carpenter dropped a hammer from a rooftop 32 feet above ground. How long did it take the hammer to hit the ground?

Height (in feet) at time t (in seconds)

$$h(t) = h_0 - 16t^2$$

where h_0 is the object's initial height (in feet)

$h(t) = 32 - 16t^2$

$-32 = -16t^2$ $\quad$ $t = \sqrt{2}$ sec

$\sqrt{2} = \sqrt{t^2}$

5. An acorn fell from a branch 45 feet high and landed on a branch 8 feet high. How long did it take the acorn to reach the branch?

$8 = 45 - 16t^2$

$-37 = -16t^2$

$\sqrt{t^2} = \sqrt{\frac{37}{16}}$

$t = \sqrt{37/16}$

Solve each quadratic equation by factoring.

6. $x^2 + 7x + 12 = 0$

$(x+3)(x+4)=0$

$x=-3 \quad x=-4$

7. $x^2 - 3x = 18$

$x^2-3x-18=0$

$(x-6)(x+3)=0$

$x=6 \quad x=-3$

8. $x^2 - 64 = 0$

$\sqrt{x^2}=\sqrt{64}$

$x=\pm 8$

3.2 Complex Numbers

9. What is the value of imaginary unit, i?

For 10-13, simplify each radical.

10. $\sqrt{-49}$

$7i$

11. $\sqrt{-27}$

$3i\sqrt{3}$

12. $\sqrt{-75}$

$5i\sqrt{3}$

13. $\sqrt{-144}$

$12i$

For 14-15, add or subtract the complex numbers.

$-2-3i$

14. $(-7 + 2i) + (5 - 11i)$

$-2-9i$

15. $(18 + 27i) - (2 + 3i)$

$16+24i$

For 16-20, simplify. *Remember:* $i^2 = -1$

16. $(4 + 9i)(6 - 2i)$

$+18$

$24-8i+54i-18i^2$

$42+46i$

17. $(12i - 3)(7 + 4i)$

$84i+48i^2-21-12i$

$72i-48-21$

$72i-69$

18. $(i\sqrt{5} + 3)(i\sqrt{5} - 3)$

$i^2 \cdot 5 - 3i\sqrt{5} + 3i\sqrt{5} - 9$

$-5-9$

~~-10~~

-14

19. $(2 + i)(2+i)$

$4+2i+2i+i^2$

-1

$3+4i$

20. $(6 + i\sqrt{2})(6 - i\sqrt{2})$

$36+2$

38

3.3 Finding Complex Solutions to Quadratic Equations

For 21-22, complete each square, then write the expression as a binomial squared.

21. $x^2 + 4x +$ ___4___

22. $x^2 - 10x +$ ___25___

For 23-24, solve by completing the square. State whether the solutions are real or non-real.

23. $x^2 - 2x + 7 = 0$

$\frac{2 \pm \sqrt{4-4(1)(7)}}{2}$ $\frac{2 \pm \sqrt{-24}}{2}$

$\frac{2 \pm \sqrt{4-28}}{2}$ $\frac{2 \pm 2i\sqrt{6}}{2}$ $1 \pm i\sqrt{6}$

non real

24. $2x^2 + 3x + 4 = 0$ →

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\frac{-3 \pm \sqrt{9-4(2)4}}{4}$ → $\frac{-3 \pm \sqrt{-23}}{4}$

$\frac{-3 \pm \sqrt{9-32}}{4}$ 4 16

non real

For 25-26, solve each equation using the quadratic formula.

25. $-5x^2 - 2x - 8 = 0$

$\frac{2 \pm \sqrt{4-4(-5)(-8)}}{-10}$ $\frac{2 \pm \sqrt{-156}}{-10}$ 156 2 78 2 39

$\frac{2 \pm \sqrt{4-160}}{-10}$ $\frac{2 \pm 2i\sqrt{39}}{-10}$ $\frac{1 \pm i\sqrt{39}}{-5}$

26. $7x^2 + 2x + 3 = -1$

$7x^2 + 2x + 4 = 0$

$\frac{-2 \pm \sqrt{4-4(7)(4)}}{14}$ → $\frac{-2 \pm \sqrt{-108}}{14}$

$\frac{-2 \pm \sqrt{4-112}}{14}$ $\frac{-2 \pm 6i\sqrt{3}}{14}$ $\frac{-1 \pm 3i\sqrt{3}}{7}$

For 27-29, use the discriminant to determine the number and type of solutions to the equation. (2 real, 1 real, or 2 non-real) $b^2 - 4ac$

27. $x^2 - 3x = -8$

$x^2 - 3x + 8 = 0$

$9 - 4(1)(8)$

$9 - 32$

-23 no solutions

28. $x^2 + 4x = -3$

$x^2 + 4x + 3 = 0$

$16 - 4(1)(3)$

$16 - 12$

4 2 real

29. $2x^2 - 12x = -18$

$2x^2 - 12x + 18 = 0$

$144 - 4(2)18$

$144 - 144$

0 1 real

58
18
136

30. A ball is thrown in the air with an initial vertical velocity of 14 m/s from an initial height of 2 m. The ball's height h (in meters) at time t (in seconds) can be modeled by the quadratic function $h(t) = -4.9t^2 + 14t + 2$.

a. Does the ball reach a height of 14m? *Write an equation and use the discriminant to answer.*

$-4.9t^2 + 14t + 2 = 14$

$-4.9t^2 + 14t - 12 = 0$

$14^2 - 4(-4.9)(-12)$

$196 - 235.2$

-39.2 no

b. How long does it take for the ball to hit the ground?

3 seconds

c. What is the maximum height of the ball can reach?

≈ 12 ft

Module 4 – Quadratic Relations and Systems of Equations

4.1 Circles

For 1-2, write the equation for the circle described.

1. $C(-3, 2)$, $r = 4$

$(x+3)^2 + (y-2)^2 = 16$

2. $C(1, -4)$, $P(-3, 5)$

$(x-1)^2 + (y+4)^2 = r^2$

$(-3-1)^2 + (5+4)^2 = r^2$

$16 + 81 = r^2$

$97 = r^2$

so $(x-1)^2 + (y+4)^2 = 97$

3. Identify the center and radius of the circle with equation $(x+2)^2 + (y+1)^2 = 36$

$(-2,-1)$ $r=6$

For 4-5, rewrite the equation of the circle in standard form. Then, identify its center and radius.

4. $x^2 + y^2 - 4x + 24y + 112 = 0$

$(x^2-4x)+(y^2+24y) = -112$

$+4 \quad +144 \quad +148$

$(x+2)^2+(x+12)^2 = 36$

Center $(2,-12)$ $r=6$

5. $2x^2 + 2y^2 - 16x - 4y + 22 = 0$

$2x^2-16x+2y^2-4y = -22$

$(x^2-8x)+(y^2-2y) = -11$

$+16 \quad +1 \quad +16 \quad +1$

$(x-4)^2+(y-1)^2 = 6$

Center $(4,1)$

$r=\sqrt{6}$

4.3 Linear-Quadratic Systems

For 6-7, solve each linear-quadratic system.

6. $\begin{cases} y = -x - 7 \\ y = x^2 - 4x - 5 \end{cases}$

$-x-7 = x^2-4x-5$

$0 = x^2-3x+2$

$0=(x-2)(x-1)$

$(x=2 \quad x=1)$

7. $\begin{cases} y = x^2 - x - 90 \\ y = x + 30 \end{cases}$

$x+30 = x^2-x-90$

$0 = x^2-2x-120$

$0=(x-12)(x+10)$

$(x=12 \quad x=-10)$

4.4 Systems of Three Linear Equations

For 8-9, solve each system algebraically.

8. $\begin{cases} -2x + y + 3z = 20 & 1 \\ -3x + 2y + z = 21 & 2 \\ 3x - 2y + 3z = -9 & 3 \end{cases}$

9. $\begin{cases} x + 2y + 3z = 9 & 1 \\ x + 3y + 2z = 5 & 2 \\ x + 4y - z = -5 & 3 \end{cases}$

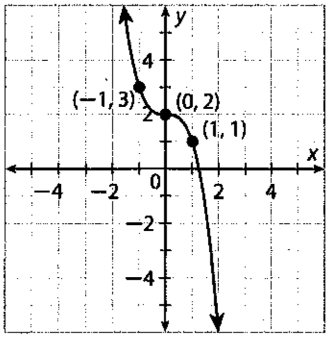
Unit 3 – Polynomial Functions, Expressions, and Equations

Module 5 – Polynomial Functions

5.1 – Graphing Cubic Functions

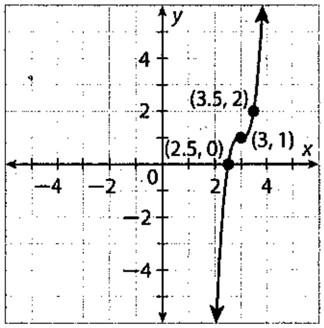
Given the general equation $f(x) = a\left(\frac{1}{b}(x-h)\right)^3 + k$, write the specific equation for the graph.

1.



$f(x) = -1(x)^3+2$

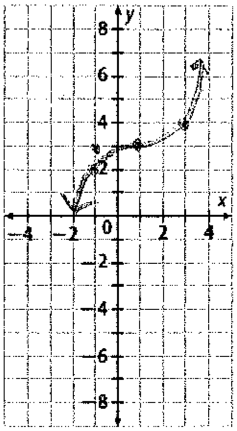
2.



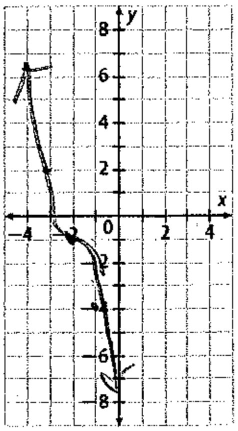
$f(x) = (2(x-3))^3+1$

For 3-4, tell what transformations have been applied to the graph of $f(x) = x^3$ to produce the graph of *g(x).* Then, graph *g(x)* by finding the point of symmetry and at least one point on each side.

3. $g(x) = \left(\frac{1}{2}(x-1)\right)^3 + 3$



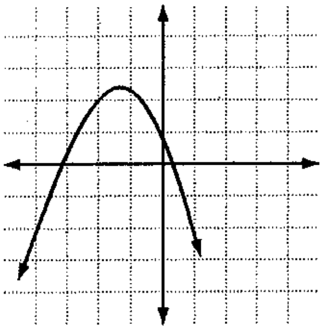
4. $g(x) = -3(x+2)^3 - 1$



5.2- Graphing Polynomial Functions

For 5-7, identify whether each function graphed has an odd or even degree and a positive or negative leading coefficient.

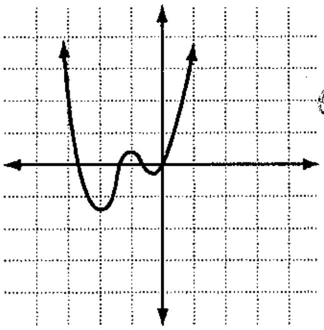
5.



even

−

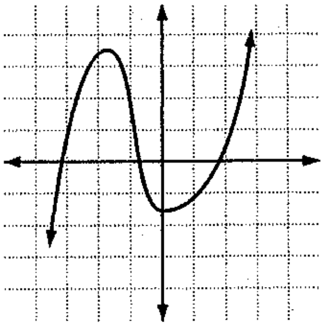
6.



even

+

7.



odd

+

For 8-9, graph each function without a calculator. State the degree, end behavior, x- and y- intercepts, and the intervals where the function is positive or negative.

8. $f(x) = -(x-1)^2(x+3)$

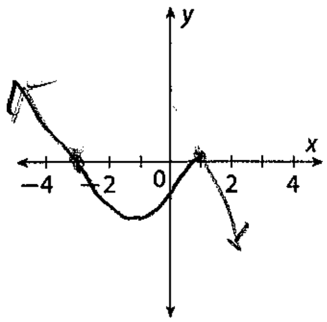
Degree: 3

x – intercept(s): 1, -3

y – intercept: (0, -3)

Positive: (-∞, -3)

Negative: (-3, 1) (1, ∞)



9. $f(x) = x(x+2)(x-3)(x-1)$

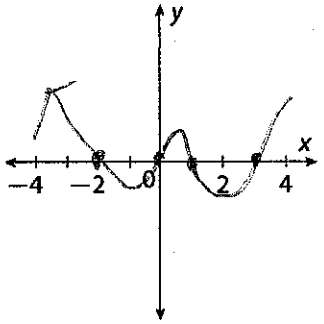
Degree: 4

x – intercept(s): 0, -2, 3, 1

y – intercept: 0

Positive: (-∞, -2) ∪ (0, 1) ∪ (3, ∞)

Negative: (-2, 0) ∪ (1, 3)



For 10-11, graph each function on a graphing calculator to determine the number of turning points, the number of global maximum and/or minimum values, and the number of local maximum and/or minimum values that are not global.

10. $f(x) = x(x-4)^2$

Turning Points: ____________

Global Maximum(s): _____ Global Minimum(s): _____

Local Maximum(s): _____ Local Minimum(s): _____

11. $f(x) = -x^2(x-2)(x+1)$

Turning Points: ____________

Global Maximum(s): _____ Global Minimum(s): _______

Local Maximum(s): _____ Local Minimum(s): ________

Module 6- Polynomials

6.1 - Adding and Subtracting Polynomials

For 1-2, perform the indicated operation. Write your answers in standard form.

$g(x) = (3x^3 - 6x - 4 + 9x^2)$ $h(x) = (2x^2 - 2x + 6)$ $k(x) = (11x^3 - x^2 - 2 + 5x)$

1. $h(x) + k(x)$

$2x^2-2x+6+11x^3-x^2-2+5x$

$\boxed{11x^3+x^2+3x+4}$

2. $h(x) - g(x)$

$2x^2-2x+6-3x^3+6x+4-9x^2$

$\boxed{-3x^3-7x^2+4x+10}$

6.2 - Multiplying Polynomials

For 3-4, perform the indicated operation. Write your answers in standard form.

$l(x) = x + 2$ $m(x) = y^2 + 2y - 12$ $n(x) = 4x^2$

3. $n(x) \cdot m(x)$ $4x^2(y^2+2y-12)$

$4x^2y^2+8x^2y-48x^2$

4. $l(x) \cdot m(x)$ $(x+2)(y^2+2y-12)$

$y^2x+2yx-12x+2y^2+4y-24$

6.3 - The Binomial Theorem

For 5-6, use the Binomial Theorem to expand each binomial.

5. $(x+y)^4$ x^4+4x^3y

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

6. $(2x-y)^3$

$8x^3-12x^2y+6xy^2-y^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

For 7-8, find the specific term of each expansion.

7. 3rd term of $(x+3y)^4$

$6x^2(9y^2)$

$54x^2y^2$

8. 2nd term of $(-3x+1)^5$

$5(-3x)^4(1)^1$

$5(81x^4)$

$405x^4$

6.4 - Factoring Polynomials

9. Complete each of the polynomial identity:

a. Difference of two squares: $a^2-b^2 = (a-b)(a+b)$

b. Perfect square trinomials: $(a+b) = a^2+2ab+b^2$

c. Sum of two cubes: $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

d. Difference of two cubes: $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

For 10-15, fully Factor.

10. $2x^2 - 10x - 48$

$2(x^2-5x-24)$

$2(x-8)(x+3)$

11. $4x^3 - 100x$

$4x(x^2-25)$

$4x(x-5)(x+5)$

12. $x^3 - 64$

$(x-4)(x^2+4x+16)$

13. $16x^2 - 8x + 1$

$(4x-1)^2$

14. $3x^2 - 48$

$3(x^2-16)$

$3(x-4)(x+4)$

15. $3x^2 - 75x$

$3x(x-25)$

For 16-17, factor each polynomial by grouping.

16. $(5x^3 - 6x^2)\overset{+}{(-} 15x + 18)$

$x^2(5x-6)-3(5x-6)$

$(x^2-3)(5x-6)$

17. $(9r^3 + 3r^2)(- 21r - 7)$

$3r^2(3r+1)-7(3r+1)$

$(3r^2-7)(3r+1)$

6.5 - Dividing Polynomials

For 18-19, divide using long division. Write the result in $divident = (divisor)(quotient) + remiander$.

18. $(x^2 - x - 6) \div (x - 3)$

$$\begin{array}{r} x+2 \\ x-3\,\overline{)\,x^2-x-6} \\ -(x^2-3x) \\ \hline 2x-6 \\ 2x-6 \end{array}$$

$(x^2-x-6) = (x-3)(x+2)$

19. $(2x^3 - 10x^2 + x - 5) \div (x - 5)$

$$\begin{array}{r} 2x^2+1 \\ x-5\,|\,2x^3-10x^2+x-5 \\ 2x^3-10x^2 \\ \hline x-5 \\ x-5 \end{array}$$

$2x^3-10x^2+x-5=(x-5)(2x^2+1)$

For 20-21, divide using synthetic division. Write the result in $\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}$.

20. $(3x^3 - 8x^2 + 4x - 6) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 3 & -8 & 4 & -6 \\ & \downarrow & 6 & -4 & 0 \\ \hline & 3 & -2 & 0 & \boxed{-6} \end{array}$$

$\frac{3x^3-8x^2+4x-6}{x-2} = 3x^2-2x+\frac{-6}{x-2}$

21. $(x^4 + 2x^3 - 3x + 1) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -3 & 1 \\ & \downarrow & -3 & 3 & 0 \\ \hline & 1 & -1 & 0 & 1 \end{array}$$

$x^2-x+\frac{1}{x+3}$

For 22-23, determine whether the given binomial is a factor of the polynomial *p(x)*. If it is, find the remaining factors of *p(x)* and write *p(x)* in factored form.

22. $p(x) = x^3 + x^2 - 10x + 8; x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -10 & 8 \\ & \downarrow & 2 & 6 & -8 \\ \hline & 1 & 3 & -4 & \boxed{0} \end{array}$$

$(x-2)(x^2+3x-4)$

$(x-2)(x+4)(x-1)=p(x)$

23. $p(x) = x^3 - \frac{1}{2}x^2 - 36x + 18; x - \frac{1}{2}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 1 & -\frac{1}{2} & -36 & 18 \\ & \downarrow & \frac{1}{2} & 0 & -18 \\ \hline & 1 & 0 & -36 & \boxed{0} \end{array}$$

$(x-1/2)(x^2-36)$

$p(x)=(x-1/2)(x+6)(x-6)$

Module 7 – Polynomial Equations

7.1 Finding Rational Solutions of Polynomial Equations

For 1-2, list all possible rational zeros/roots for the equation. Find the actual zeros/roots of the function and write the function in factored form.

$\pm 1, 2, 3, 4, 6, 8, 12, 16, 24, 48$

1. $f(x) = x^3 + 5x^2 - 8x - 48$

$3|\ 1 \quad 5 \quad -8 \quad -48$

$1 \quad 3 \quad 24 \quad 48$

$1 \quad 8 \quad 16 \quad \boxed{0}$

$(x-3)(x^2+8x+16)$

$(x-3)(x+4)(x+4)$

2. $p(x) = 2x^4 + x^3 - 19x^2 - 9x + 9$

$\frac{\pm 1, 3, 9}{\pm 1, 2}$

$-3|\ 2 \quad 1 \quad -19 \quad -9 \quad 9$

$1 \quad -6 \quad 15 \quad 12 \quad -9$

$3|\ 2 \quad -5 \quad -4 \quad 3 \quad \boxed{0}$

$1 \quad 6 \quad 3 \quad -3$

$2 \quad 1 \quad -1 \quad \boxed{0}$

$(x-3)(x+3)(2x^2+x-1)$

$p(x) = (x-3)(x+3)(2x-1)(x+1)$

For 3-6, solve each polynomial equation.

3. $4x^4 - 64x^2 = 0$ $\quad x=0 \quad x=4 \quad x=-4$

$4x^2(x^2-16)=0$

$4x^2(x-4)(x+4)=0$

4. $(x^3 + 3x^2)(- x - 3) = 0$ $\quad (x^2-1)(x+3)=0$

$x^2(x+3)-1(x+3)=0$ $\quad (x+1)(x-1)(x+3)=0$

$x=-1 \quad x=1 \quad x=-3$

5. $x^4 - 14x^2 + 45 = 0$

$(x^2-9)(x^2-5)=0$

$x^2-9=0$

$x=\pm 3$

$x^2-5=0 \quad x^2=5$

$x=\pm\sqrt{5}$

6. $x^4 - 7x^3 - 3x^2 + 63x - 54 = 0$

For 7-8, solve each equation by factoring.

7. $(4x^3 + x^2)(- 4x - 1) = 0$

$x^2(4x+1)-1(4x+1)=0$

$(x^2-1)(4x+1)=0$

$(x+1)(x-1)(4x+1)=0$

$x=-1, 1, -1/4$

8. $x^5 - 2x^4 - 24x^3 = 0$ $\quad x=0, 6, -4$

$x^3(x^2-2x-24)=0$

$x^2(x-6)(x+4)=0$

7.2 Finding Complex Solutions of Polynomial Equations

For 9-11, solve each equation by finding all roots.

9. $(x^3 - 2x^2) + (3x - 6) = 0$

$x^2(x-2)+3(x-2)=0$

$(x^2+3)(x-2)=0$

$x=\pm\sqrt{3} \quad x=2$

10. $x^3 + 3x^2 - 14x - 20 = 0$

Possible roots

$\pm 1, 2, 4, 5, 10, 20$

$-5|\ 1 \quad 3 \quad -14 \quad -20$

$1 \quad -5 \quad 10 \quad 20$

$1 \quad -2 \quad -4 \quad \boxed{0}$

$(x-5)(x^2-2x-4)$

$x=5$

$x=\frac{2\pm\sqrt{4-4(1)(-4)}}{2}$

$x=\frac{2\pm\sqrt{4+16}}{2}$

$x=\frac{2\pm\sqrt{20}}{2}$

$x=\frac{2\pm 2\sqrt{5}}{2}$

$x=1+\sqrt{5}$

11. $(x^3 - 3x^2) + (4x - 12) = 0$ $\quad x=\pm 2$

$x^2(x-3)+4(x-3)=0$ $\quad x=3$

$(x^2+4)(x-3)=0$

For 12-13, write the simplest polynomial function with the given roots.

12. $1, 4, \text{and} -3$

$(x-1)(x-4)(x+3)$

$(x^2-5x+4)(x+3)$

$(x^3+3x^2-5x^2-15x+4x+12)$

$x^3-2x^2-11x+12$

13. $2i$ and $\sqrt{3}$ $\quad -2i \quad -\sqrt{3}$

$(x-2i)(x+2i)(x-\sqrt{3})(x+\sqrt{3})$

$(x^2+4)(x^2-3)$

$x^4-3x^2+4x^2-12$

x^4+x^2-12

Unit 4 – Rational Functions, Expressions and Equations

Module 8- Rational Functions

8.1 Graphing Simple Rational Functions

For 1-2, identify the asymptotes, domain and range in set notation for each function.

1. $g(x) = \frac{1}{x-3} + 5$

VA $X=3$ Domain $\{X|X=3\}$

HA $Y=5$ Range $\{y|y=-5\}$

2. $g(x) = \frac{1}{x+8} - 1$

VA $X=-8$ Domain $\{X|X\neq 8\}$

HA $Y=-1$ Range $\{y|y\neq -1\}$

3. Rewrite the function in $g(x) = a\left(\frac{1}{(x-h)}\right) + k$ or $g(x) = \left(\frac{1}{\frac{1}{b}(x-h)}\right) + k$ form, then find the asymptotes and domain and range and sketch the graph of $g(x)$.

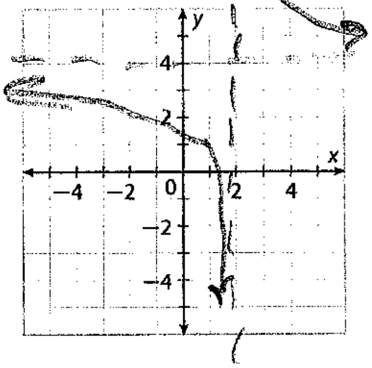
$$g(x) = \frac{4x-5}{x-2}$$

$$x-2\,\overline{)\,4x-5}$$ quotient 4, $-(4x-8)$, remainder 3

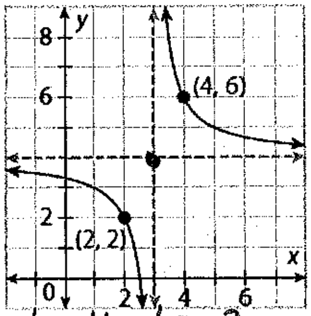
$g(x) = 4 + \frac{3}{x-2}$

$g(x) = \frac{3}{x-2} + 4$

$g(x) = 3\left(\frac{1}{x-2}\right) + 4$



4. Write the equation of the graph in the format of $f(x) = a\left(\frac{1}{(x-h)}\right) + k$ or $f(x) = \left(\frac{1}{\frac{1}{b}(x-h)}\right) + k$



$h=3 \quad k=4$

$(4,6)$ $h=3$ $k=3$

$6 = \left(\frac{1}{\frac{1}{b}(4-3)}\right) + 4$

$2 = \left(\frac{1}{\frac{1}{b}(-1)}\right)$

$2 = \left(\frac{1}{-\frac{1}{b}}\right)$

$2 = -b$

$\boxed{b=-2}$

$h=3 \quad k=4 \quad b=-2$

$f(x) = -2\left(\frac{1}{x-3}\right) + 4$

8.2 Graphing More Complicated Rational Functions

For 5-6, identify all vertical asymptotes, holes, and horizontal or slant asymptote of each rational function. Then state its domain.

5. $f(x) = \frac{x-1}{-3x^2+27} = \frac{(x-1)}{-3(x^2-9)} = \frac{(x-1)}{-3(x-3)(x+3)}$

6. $f(x) = \frac{-x^2-3x+4}{x^2+2x-8} = \frac{-(x+4)(x-1)}{(x+4)(x-2)}$ $\quad -(x^2+3x-4)$

Vertical Asymptotes: $X=3 \quad X=-3$

Horizontal or Slant Asymptote: $y=0$

Holes: no

Domain: $\{x|\ x\neq 3\ x\neq -3\}$

Vertical Asymptotes: $X=2$

Horizontal or Slant Asymptote: $y=-1$

Holes: $X=-4$

Domain: $\{x|\ x\neq -4\ x\neq 2\}$

$-\frac{1}{3}x+\frac{1}{3}$

x^2-0x-4

$-(x^2+x)$

$-x-4$

7. $f(x)=\frac{x^2-4}{-3x}$ $=\frac{(x-2)(x+2)}{-3x}$ $-3x$

Vertical Asymptotes: $x=0$

Horizontal or Slant Asymptote: $y=-\frac{1}{3}x+\frac{1}{3}$

Holes: no

Domain: $\{x \mid x\neq 0\}$

8. $f(x)=\frac{-2x+1}{x-2}$

Vertical Asymptotes: $x=2$

Horizontal or Slant Asymptote: $y=-2$

Holes: non

Domain: $\{x\} \ x\neq 2\}$

Module 9 – Rational Expressions and Equations

9.1 Adding and Subtracting Rational Expressions

For 1-2, add or subtract. Identify any excluded values.

1. $\frac{x+4}{x^2-x-12}+\frac{2x}{x-4}\frac{(x+3)}{(x+3)}$

$(x-4)(x+3)$

$\frac{x+4+2x^2+6x}{(x-4)(x+3)}=\frac{2x^2+7x+4}{(x-4)(x+3)}$

2. $\frac{3x^2-1}{x^2-3x-18}+\frac{-(x+2)}{x-6}\frac{(x+3)}{(x+3)}$

$(x-6)(x+3)$

$\frac{3x^2-1+(-x-2)(x+3)}{(x-6)(x+3)}$

$\frac{3x^2-1-x^2-5x-6}{(x-6)(x+3)}=\frac{2x^2-5x-7}{(x-6)(x+3)}$

$\frac{(x+2)}{(x+2)}$ 3. $\frac{2x-3}{1}+\frac{1}{x+2}$

$\frac{2x^2+x-6+1}{x+2}$

$\frac{2x^2+x-5}{x+2}$

9.2 Multiplying and Dividing Rational Expressions

For 3-4, multiply. Identify any excluded values.

3. $\frac{1}{x+9}\cdot\frac{7x^3+49x^2}{x+7}=\frac{1}{x+9}\cdot\frac{7x^2\cancel{(x+7)}}{\cancel{x+7}}$

$\frac{7x^2}{x+9}$

4. $\frac{6x^2-54x}{\cancel{x-9}}\cdot\frac{1}{\cancel{2}}$ $\cancel{6x}\cancel{(x-9)}$ $=3x$

For 5-6, divide. Identify any excluded values.

5. $\frac{6(x-2)}{(x-10(x-10)}\div\frac{x-2}{x-10}$

$\frac{6\cancel{(x-2)}}{(x-10)\cancel{(x-10)}}\cdot\frac{\cancel{x-10}}{\cancel{(x-2)}}$ $\frac{6}{x-10}$

6. $\frac{27x+9}{10}\div\frac{3x^2-8x-3}{10}$

$\frac{9\cancel{(3x+1)}}{\cancel{10}}\cdot\frac{\cancel{10}}{\cancel{(3x+1)}(x-3)}=\frac{9}{x-3}$

9.3 Solving Rational Equations

Solve each equation algebraically.

$6x$ 7. $\left(\frac{1}{x}-\frac{x-2}{2x}\right)=\frac{4}{3}$ $\cdot 6x$

$\left(6x\cdot\frac{1}{x}\right)-\left(\frac{6x}{1}\cdot\frac{x-2}{2x}\right)=\frac{4}{3}\cdot\frac{6x}{1}$

$6-3(x-2)=8x$

$6-3x+6=8x$

$-3x+12=8x$ $x=\frac{12}{11}$

$12=11x$

x 8. $\left(\frac{x^2-7x+10}{x}+\frac{1}{x}\right)=(x+4)x$

$x^2-7x+10+1=x^2+4x$

$-x^2$ $-x^2$

$-7x+11=4x$

$11=11x$

$x=1$

(x^2-4) 9. $\frac{4}{x^2-4}=\frac{1}{x-2}$ $(x-2)(x+2)$

$4=x+2$

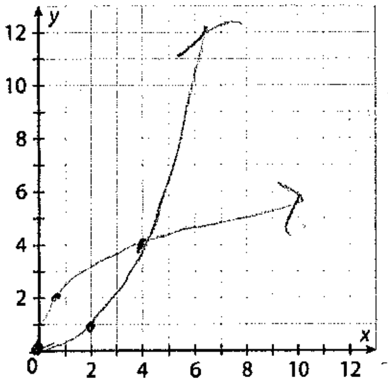
$x=2$

10.1 Inverses of Simple Quadratic and Cubic Functions

For 1-2, graph the function *f(x)* for the domain $\{x|x \geq 0\}$. Then write and graph its inverse function, $f^{-1}(x)$.

1. $f(x) = 0.25x^2$

$F(x) = \frac{1}{4}x^2$



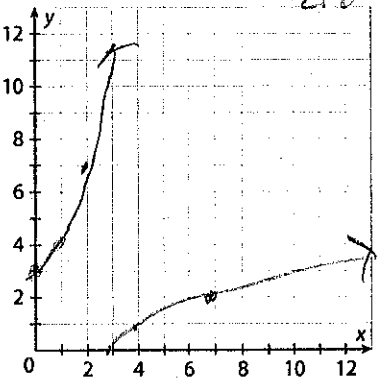
x	f(x)
0	0
2	1
4	4

$f^{-1}(x) = 2\sqrt{x}$

x	$f^{-1}(x)$
0	0
1	2

2. $f(x) = x^2 + 3$

x	f(x)
0	3
1	4
2	7



$x = y^2 + 3$

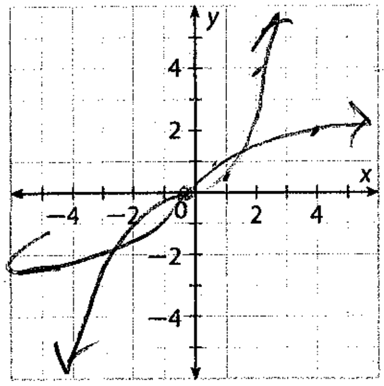
$\sqrt{x-3} = y$

$f^{-1}(x) = \sqrt{x-3}$

x	$f^{-1}(x)$
7	2
4	1
3	0

For 3-4, graph the function *f(x)*. Then write and graph its inverse function, $f^{-1}(x)$.

3. $f(x) = 0.5x^3$

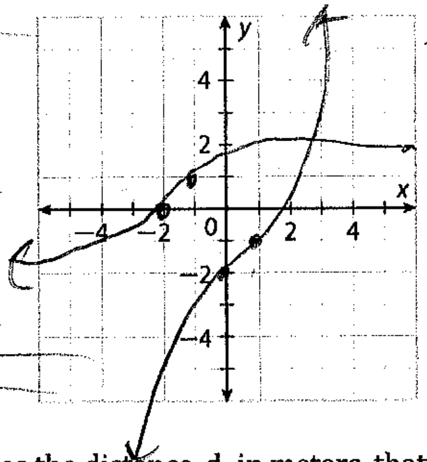


x	f(x)
0	0
1	1/2
2	4

$f^{-1}(x) = \sqrt[3]{2x}$

x	$f^{-1}(x)$
4	2
1/2	1
0	0

4. $f(x) = x^3 - 2$



x	f(x)
0	-2
1	-1
2	6

$f^{-1}(x) = \sqrt[3]{x+2}$

x	$f^{-1}(x)$
-2	0
-1	1
6	2

For 5-6, use the function $d(t) = 4.9t^2$ which gives the distance, d, in meters, that an object dropped from a height will fall in t seconds.

5. Write its inverse function *t(d)* for the time, t, in seconds it takes for an object to fall a distance of d meters.

$\frac{d}{4.9} = \frac{4.9t^2}{4.9}$ $\quad \sqrt{t^2} = \sqrt{\frac{d}{4.9}}$ $\quad t = \sqrt{\frac{d}{4.9}}$

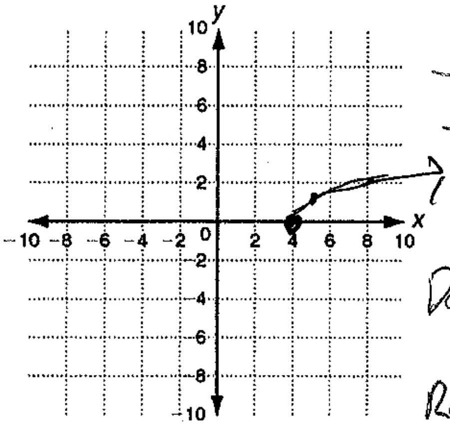
6. Find the number of seconds it takes an object to fall 150 meters. Round to the nearest 10th of a second.

$t = \sqrt{\frac{150}{4.9}}$ use calculator

10.2 Graphing Square Root Functions

For 1-2, find the endpoint and two additional points to graph each function. Identify the domain and range.

1. $f(x) = \sqrt{x-4}$

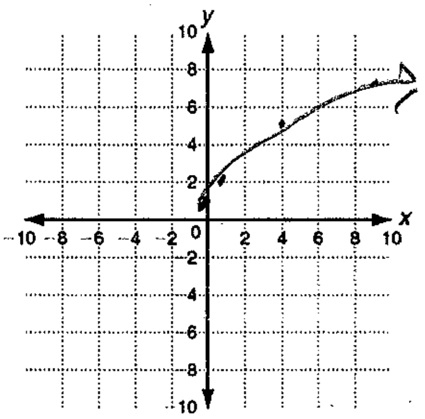


x	f(x)
4	0
5	1
8	2

Domain
$\{x \mid x \geq 4\}$

Range
$\{y \mid y \geq 0\}$

2. $f(x) = 2\sqrt{x} + 1$

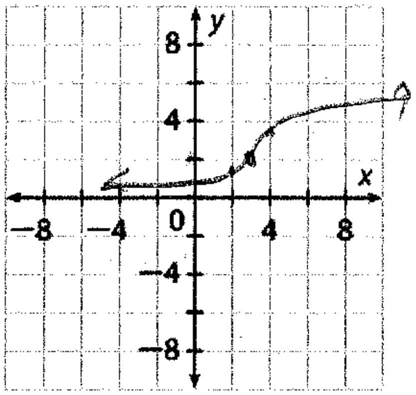


x	f(x)
0	1
1	3
4	5
9	7

10.3 Graphing Cube Root Functions

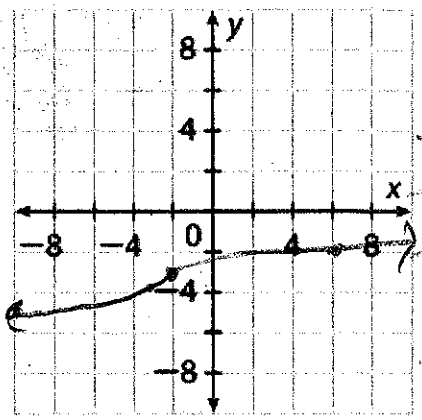
For 1-2, tell the transformations that have been applied to the parent graph of $f(x) = \sqrt[3]{x}$ to produce the graph of $g(x)$. Then graph each cube root function by finding the point of symmetry and two points on each side.

1. $g(x) = \sqrt[3]{x-3} + 2$



x	f(x)
3	2
4	3
2	-3

2. $g(x) = \frac{1}{2}\sqrt[3]{x+2} - 3$



x	f(x)
-10	-5
-3	-3.5
-2	-3
6	-2

Module 11- Radical Expressions and Equations

11.1 Radical Expressions and Rational Exponents

For 1-3, translate the expression with rational exponents into a radical expression and simplify, if possible.

1. $X^{\frac{10}{3}}$ $\sqrt[2]{X^{10}}$

2. $(81x^5y^3)^{\frac{1}{4}}$

$3x^{5/4}y^{3/4}$

3. $(27)^{\frac{2}{3}}$ $\sqrt[3]{27}^{2}$

$3^{2}\ \boxed{9}$

For 4-7, translate the radical expression into an expression with rational exponents and simplify, if possible.

4. $\sqrt[3]{x^5}$ $\sqrt[3]{x}^{5}$

5. $\sqrt{18x^7}$ $3x^3\sqrt{2x}$

6. $\sqrt[3]{27^2}$ $3^2=9$

7. $\sqrt[5]{(-32)^2}$ $(-2)^2=4$

2 16
2 8
2 4
2 2

11.2 Simplifying Radical Expressions

For 8-13, simplify the expression. Assume that all variables are positive. All exponents should be positive in simplified form. Rationalize any irrational denominators.

8. $\frac{\sqrt{a^3b}}{a^2b^2}$ $\frac{a^{3/2}b^{1/2}}{a^2b^2}$ $\boxed{\frac{1}{a^{1/2}b^{3/2}}}$

9. $\frac{\sqrt[4]{36}\cdot\sqrt[4]{216}}{\sqrt[4]{6}}$ $\frac{6^{1/2}\cdot 6^{3/4}}{6^{1/4}}$ $6^{1/2}\cdot 6^{1/2}=\boxed{6^1}$

10. $\sqrt{27}\cdot\sqrt{3^5}\cdot\sqrt[3]{9}=3^{3/2}\cdot3^{5/2}\cdot3^{2/3}$

$3^{8/2}\cdot3^{2/3}$

$3^4\cdot3^{2/3}$

$3^{12/3}\cdot3^{2/3}=\boxed{3^{14/3}}$

11. $\frac{2x^2y^3}{6x^{\frac{2}{3}}}=\frac{x^{4/3}y^3}{x^{2/3}}$ $\boxed{x^{4/3}y^3}$

12. $(\frac{x^{\frac{4}{3}}}{16x^{\frac{2}{3}}})^{\frac{3}{2}}$ $\frac{x^2}{16^{3/2}x^1}=\boxed{\frac{x}{64}}$

13. $\frac{a^{-2}\cdot a^{\frac{2}{3}}}{a^{\frac{3}{4}}}=\frac{a^{2/3}}{a^2a^{3/4}}$ $\frac{a^{8/12}}{a^{24/12}a^{9/12}}=\boxed{\frac{1}{a^{25/12}}}$

11.3 Solving Radical Equations

For 14-17, solve each equation. Identify any extraneous roots.

14. $(4x+7)^{\frac{1}{2}}=3$

$4x+7=9$

$4x=2$

$\boxed{x=1/2}$

15. $2+\sqrt{x-2}=x$

$-2 \quad -2$

$(\sqrt{x-2})^2=(x-2)^2$

$x-2=x^2-4x+4$

$-x+2 \quad -x+2$

$0=x^2-5x+6$

$0=(x-3)(x-2)$

$0=x-3 \quad 0=x-2$

$+3 \quad +3 \quad +2 \quad +2$

$\boxed{x=3}$ $\boxed{x=2}$

$2+\sqrt{3-2}=3$

$2+1=3$ ✓

$2+\sqrt{2-2}=2$

$2+0=2$ ✓

16. $(\sqrt{x+15})^2=(x-5)^2$

$x+15=x^2-10x+25$

$-x-15 \quad -x-15$

$0=x^2-11x+10$

$0=(x-10)(x-1)$

$\boxed{x=10}$ ~~$x=1$~~

$\sqrt{10+15}=10-5$

$\sqrt{25}=5$ ✓

$\sqrt{1+15}=1-5$

$\sqrt{16}\neq-4$

$\boxed{ER: x=1}$

17. $\sqrt[3]{2x-2}=6$

$2x-2=216$

$2x=218$

$\boxed{x=109}$

18. The trunk length (in inches) of a male elephant can be modeled by $l=23\sqrt[3]{t}+17$, where t is the age of the elephant in years. If a male elephant has a trunk length of 80 inches, about what is his age?

$80=23\sqrt[3]{t}+17$

$-17 \quad -17$

$\frac{63}{23}=\frac{23\sqrt[3]{t}}{23}$

$(\sqrt[3]{t})^3=(\frac{63}{23})^3$ calculate

$t=$ ______